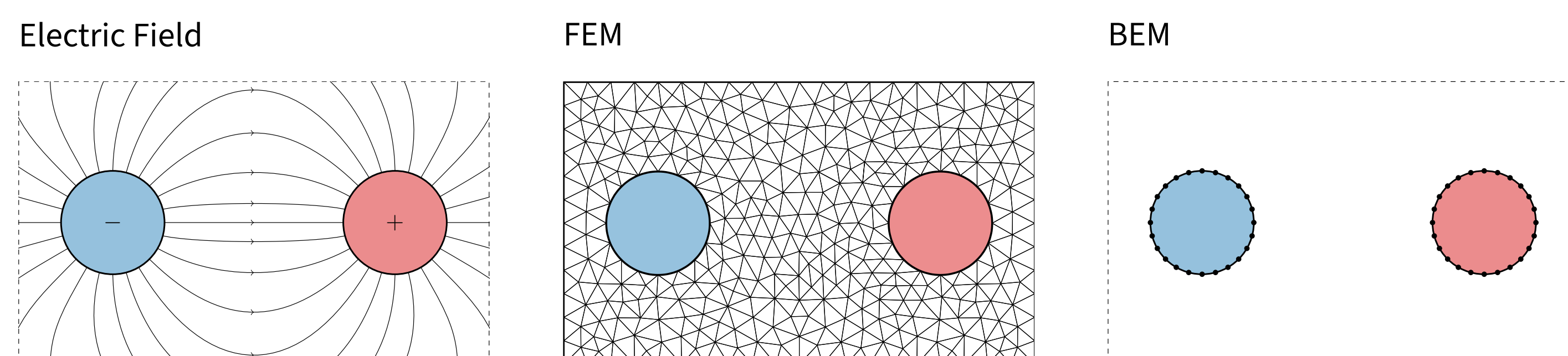




Large-scale electrostatic simulations are increasingly important for the virtual prototyping of high-voltage devices such as bushings, insulators, gas-insulated switchgears and circuit breakers. We present a distributed-memory high-performance computational framework for solving industrial electrostatic problems using the **Boundary Element Method**. To overcome its computational complexity, a multilevel **Fast Multipole Method** is combined with hybrid shared- and distributed-memory parallelization algorithms. The implementation enables virtual prototyping at scale of the aforementioned devices on modern HPC systems, requiring only a **surface mesh** for the description of the geometry.

BOUNDARY ELEMENT METHOD



Indirect Boundary Integral Formulation

- Electrostatic boundary value problem ($\mathbf{E} = -\mathbf{grad} u$)
 $-\text{div } \varepsilon_f \mathbf{grad} u = 0$

plus *boundary* and *transmission* conditions

- Floating potentials Ω_f with prescribed charge Q_f

$$-\int_{\partial\Omega_f} \varepsilon \mathbf{grad} u \cdot \mathbf{n} ds = Q_f$$

- Single layer potential

$$u(x) = \int_{\Gamma} \frac{1}{4\pi|x-y|} \varphi(y) ds_y = (S\varphi)(x)$$

- Dirichlet and Neumann traces

$$\gamma_D u = \gamma_D S\varphi = \mathcal{V}\varphi \quad \text{and} \quad \gamma_N^\pm u = \gamma_N^\pm S\varphi = \mp \frac{1}{2}\varphi + \mathcal{K}'\varphi$$

BEM System

- Physical decomposition: **electrodes**, **dielectrics** & **floating conductors**
- Boundary and transmission conditions lead to block system of matrices

$$\begin{bmatrix} \tilde{K}_{DD} & K_{DE} & K_{DF} & 0 \\ V_{ED} & V_{EE} & V_{EF} & 0 \\ V_{FD} & V_{FE} & V_{FF} & H_F \\ F_D & F_E & F_F & 0 \end{bmatrix} \begin{bmatrix} \varphi_D \\ \varphi_E \\ \varphi_F \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{g}_E \\ 0 \\ 0 \end{bmatrix}$$

Implementation

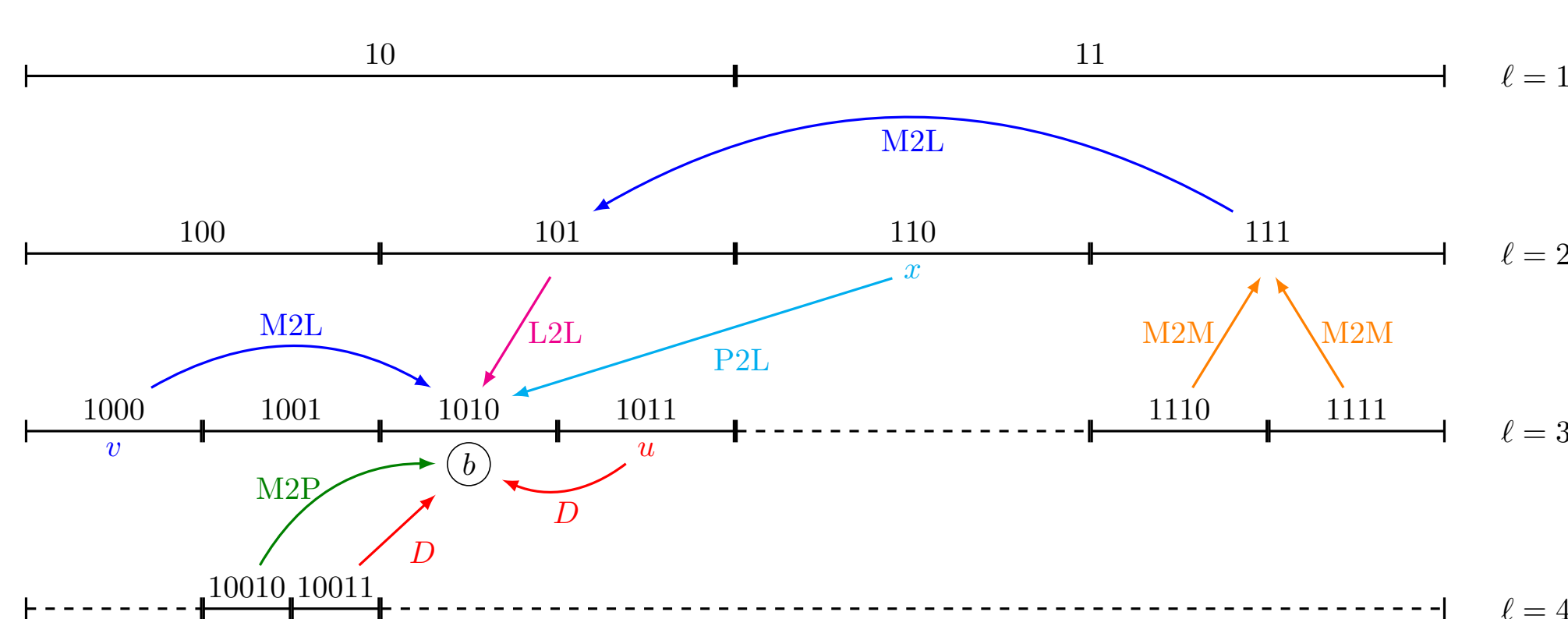
- Galerkin Boundary Element Method
- Surface approximation of orders 1, 2, and 3
- Hierarchical shape functions $\varphi(x) \approx \sum_i^N \varphi_i \psi_i^p(x)$, $p = 0, 1, 2, \dots$
- Singular integral kernels require sophisticated, robust quadratures
- Fully populated system matrices imply $\mathcal{O}(N^2)$ complexity
 $\rightarrow$ Fast Multipole Method (FMM) with $\mathcal{O}(N)$
- GMRES solver with Jacobi preconditioner

FAST MULTIPOLE METHOD

- Multipole expansion with respect to center c

$$\frac{1}{|x-y|} = \sum_{n=0}^{\infty} \sum_{m=-n}^n R_n^m(y-c) S_n^m(x-c)$$

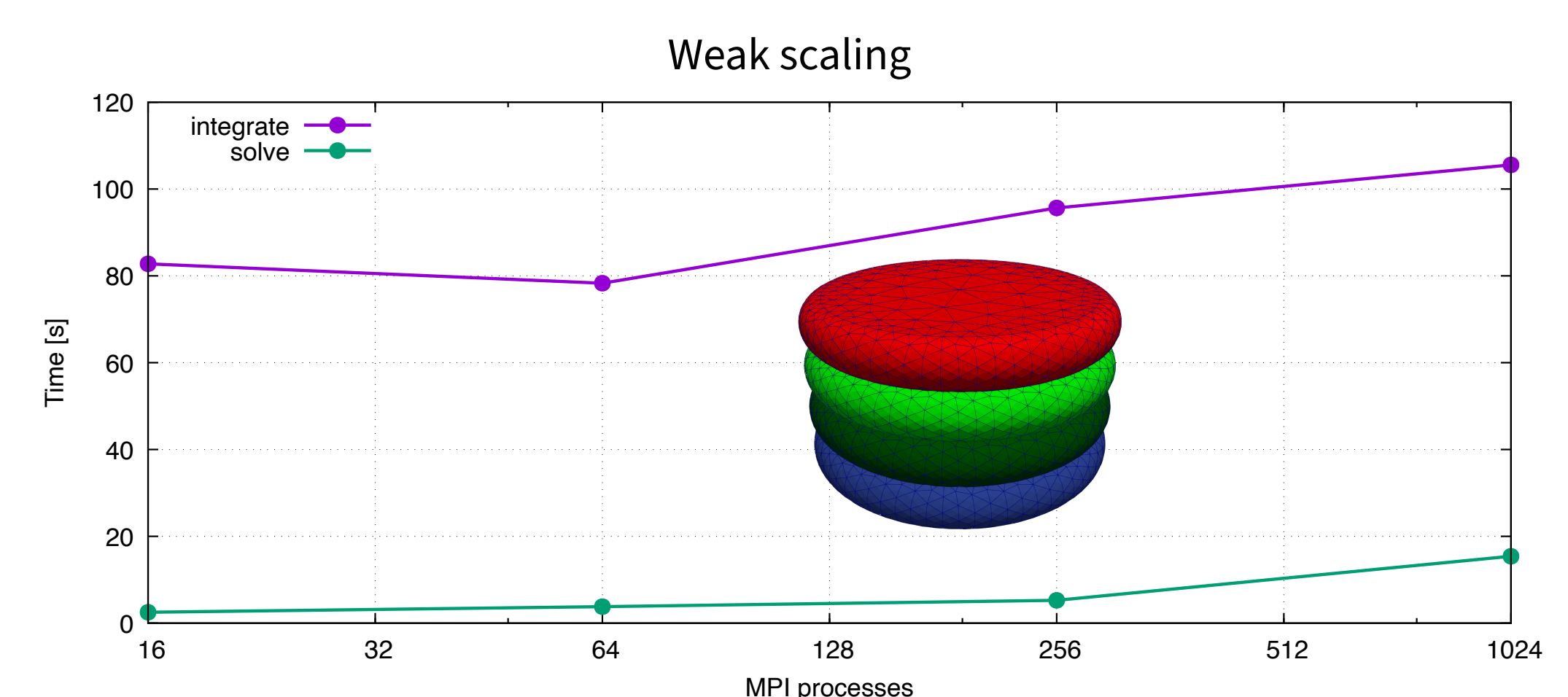
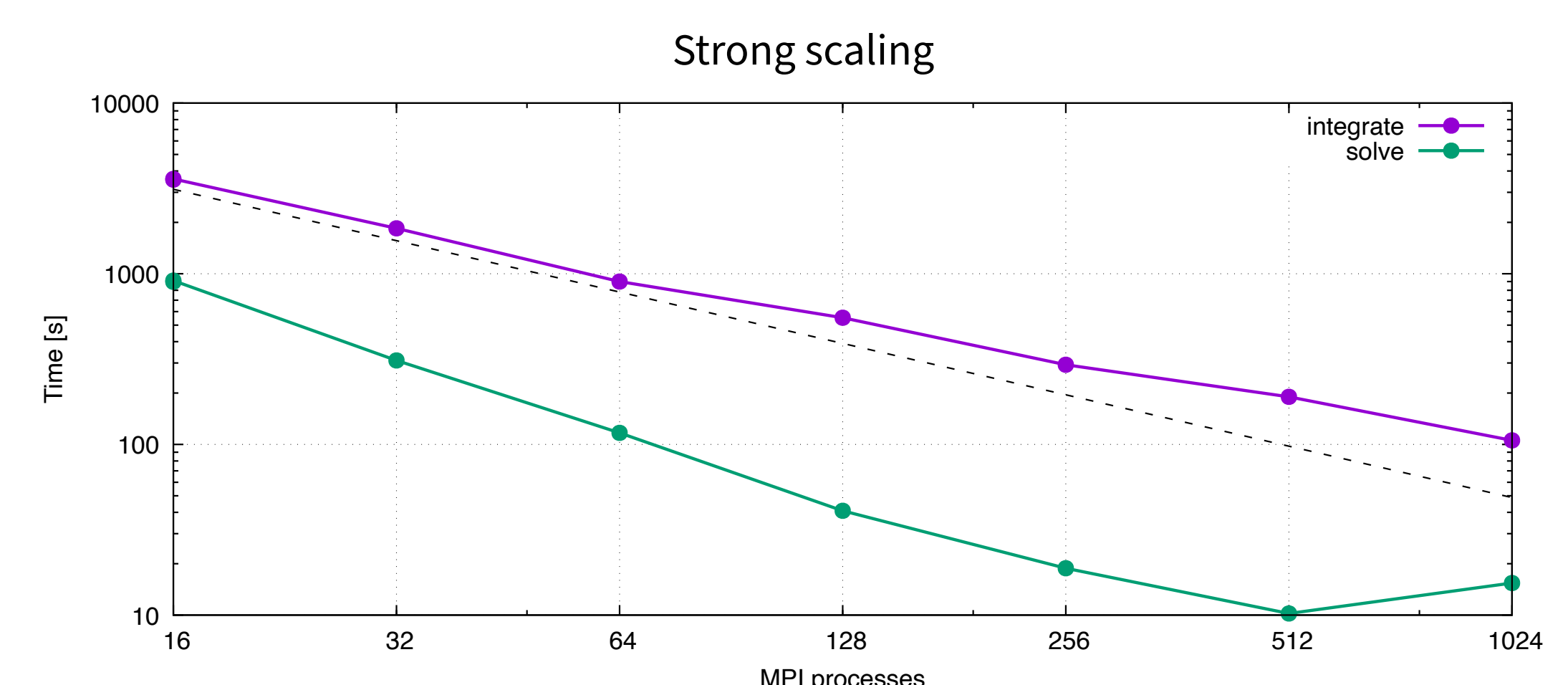
- Regular and singular solid harmonics, R_n^m and S_n^m
- Adaptive multi-level interaction operators



- MPI-parallel and multi-threaded

SCALING RESULTS

- Test model with electrode, two dielectrics and a floating potential
- Up to 3.9M surface elements on up to 1024 MPI processes



ACKNOWLEDGMENTS & REFERENCES

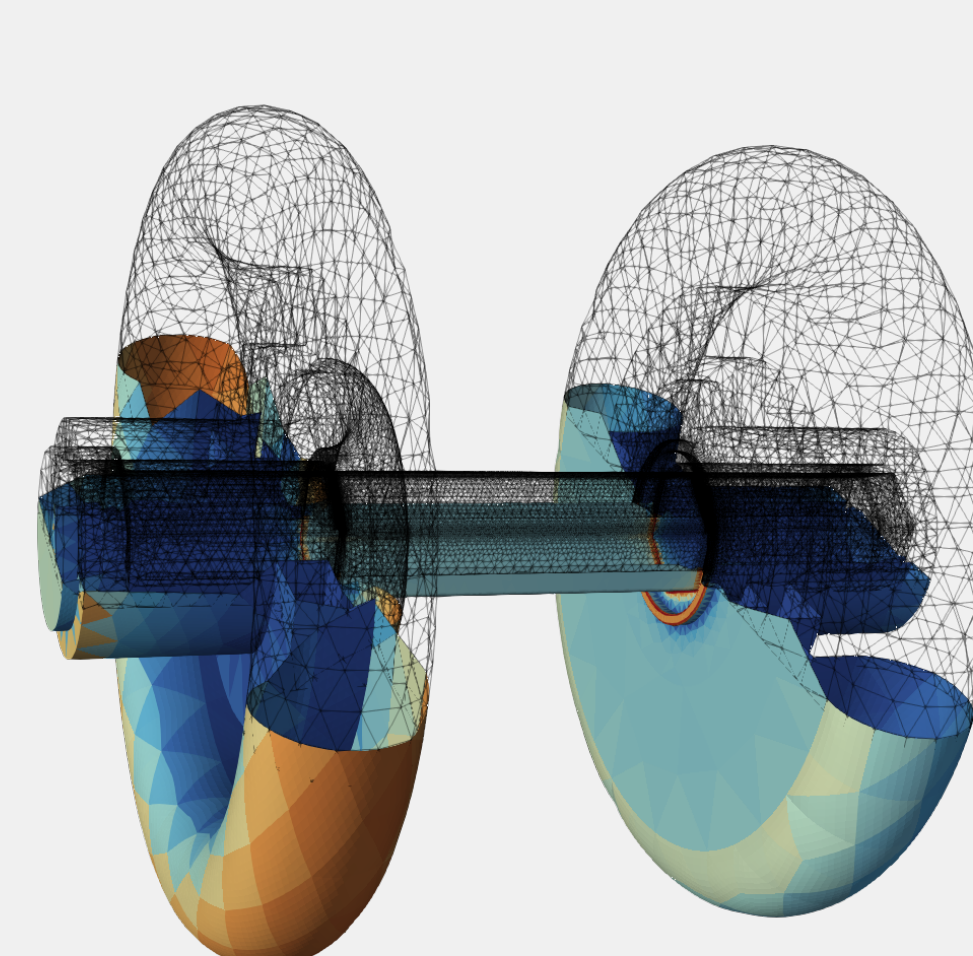
The computational results have been achieved using the Austrian Scientific Computing (ASC) infrastructure. Support in accessing the infrastructure was given by Advanced Computing Austria (ACA), <https://www.asc.ac.at>.

- A. Blaszczyk, "Region-oriented BEM formulation for numerical computations of electric fields," in *Scientific Computing in Electrical Engineering SCEE 2008*, Springer, 2010.
- D. Amann, et al. "Simulation of floating potentials in industrial applications by boundary element methods," *Journal of Mathematics in Industry*, 4.1:13, 2014.
- I. Lashuk et al., "A massively parallel adaptive fast-multipole method on heterogeneous architectures," in *Proc. Conference on HPC Networking, Storage and Analysis*, 2009.

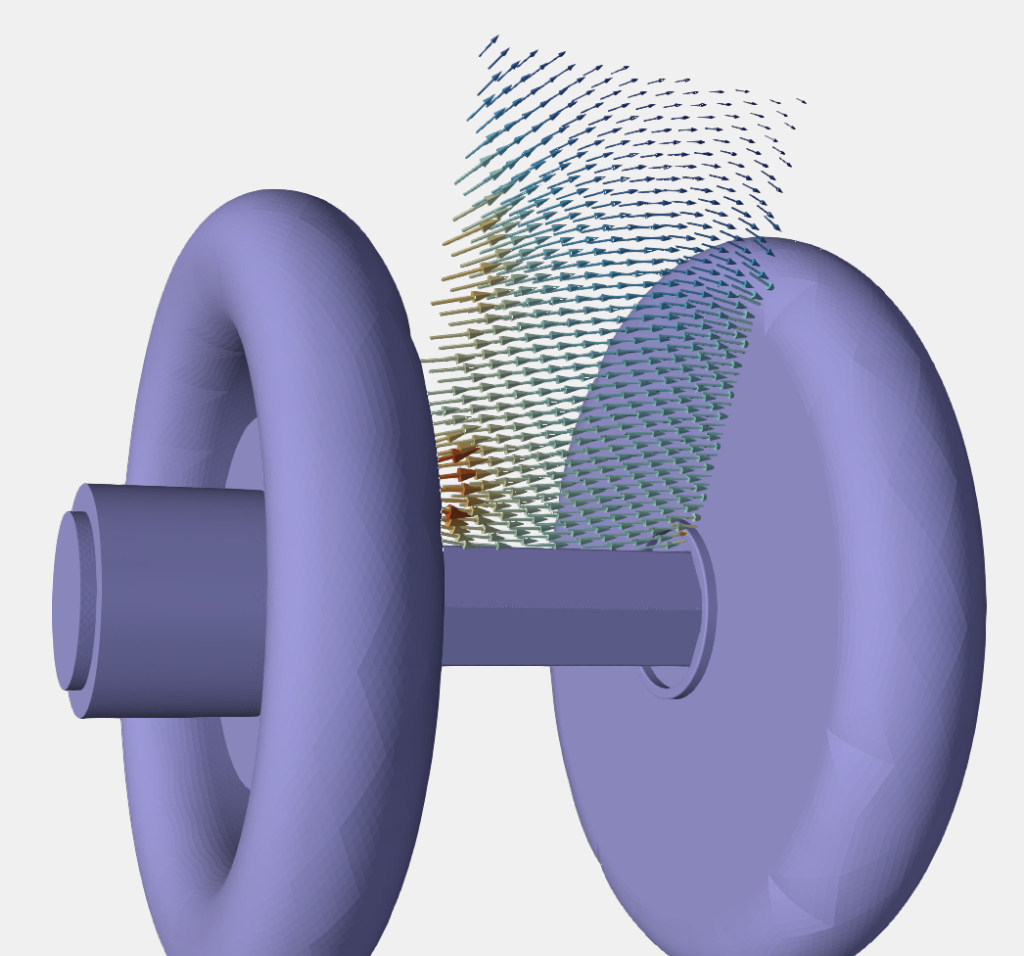
POLARIS — HPC ELECTROSTATICS SOLVER

Features

- Computation of potential and E-field on the surface *and* inside the domain using a FMM-accelerated Boundary Element Method
- Runs on Linux clusters and workstations with MPI



Computed field, 1.6M elements



Field evaluation

More information and free trial version



<https://www.tailsit.com/polaris>