Large-Scale Electrostatic Simulations with the Boundary Element Method on HPC Systems

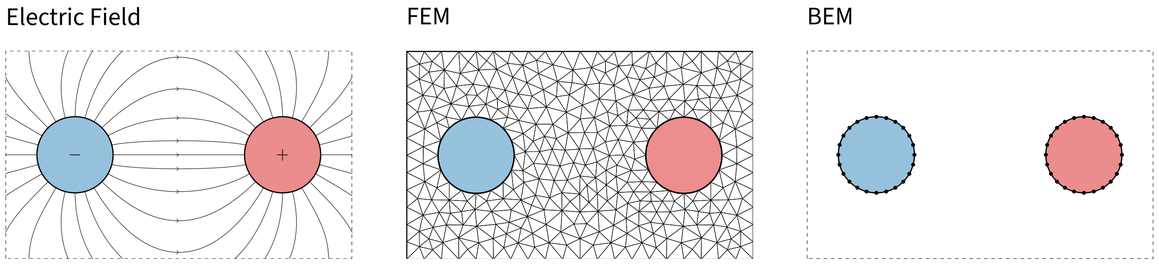
Jürgen Zechner Thomas Rüberg Lars Kielhorn

TAILSIT GmbH, Nikolaiplatz 4, 8020 Graz, Austria

IGTE'26 22nd International IGTE Symposium 2026 — September, 20 - 23, 2026, Graz, Austria

Large-scale electrostatic simulations are increasingly important for the virtual prototyping of high-voltage devices such as bushings, insulators, gas-insulated switchgears and circuit breakers. We present a distributed-memory high-performance computational framework for solving industrial electrostatic problems using the **Boundary Element Method**. To overcome its computational complexity, a multilevel **Fast Multipole Method** is combined with hybrid shared- and distributed-memory parallelization algorithms. The implementation enables virtual prototyping at scale of the aforementioned devices on modern HPC systems, requiring only a **surface mesh** for the description of the geometry.

Boundary Element Method



Indirect Boundary Integral Formulation

- Electrostatic boundary value problem ($\mathbf{E} = -\mathbf{grad}\, u$)

$$-\operatorname{div} \varepsilon_r \,\mathbf{grad}\, u = 0$$

  plus *boundary* and *transmission* conditions
- Floating potentials Ω_f with prescribed charge Q_f

$$-\int_{\partial\Omega_f} \varepsilon \,\mathbf{grad}\, u \cdot \mathbf{n} \mathrm{d}s = Q_f$$

- Single layer potential

$$u(x) = \int_\Gamma \frac{1}{4\pi|x-y|}\varphi(y)\mathrm{d}s_y = (\mathcal{S}\varphi)(x)$$

- Dirichlet and Neumann traces

$$\gamma_D u = \gamma_D \mathcal{S}\varphi = \mathcal{V}\varphi \quad \text{and} \quad \gamma_N^{\pm} u = \gamma_N^{\pm}\mathcal{S}\varphi = \mp\frac{1}{2}\varphi + \mathcal{K}'\varphi$$

BEM System

- Physical decomposition: electrodes, dielectrics & floating conductors
- Boundary and transmission conditions lead to block system of matrices

$$\begin{bmatrix} \tilde{K}_{DD} & K_{DE} & K_{DF} & 0 \\ V_{ED} & V_{EE} & V_{EF} & 0 \\ V_{FD} & V_{FE} & V_{FF} & H_F \\ F_D & F_E & F_F & 0 \end{bmatrix} \begin{bmatrix} \varphi_D \\ \varphi_E \\ \varphi_F \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ g_E \\ 0 \\ 0 \end{bmatrix}$$

Implementation

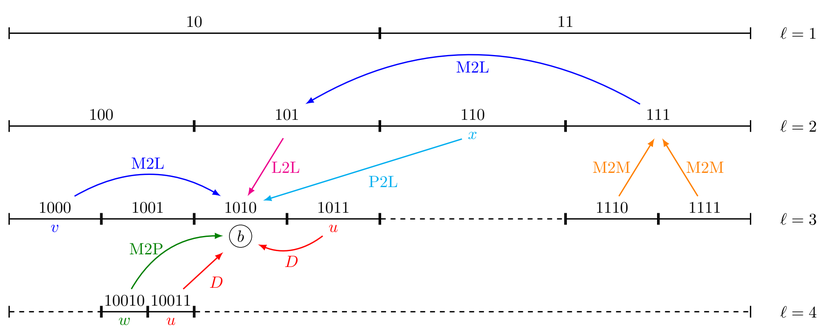
- Galerkin Boundary Element Method
- Surface approximation of orders 1, 2, and 3
- Hierarchical shape functions $\varphi(x) \approx \sum_i^N \varphi_i \psi_i^p(x), p = 0, 1, 2, \ldots$
- Singular integral kernels require sophisticated, robust quadratures
- Fully populated system matrices imply $\mathcal{O}(N^2)$ complexity
  $\rightarrow$ Fast Multipole Method (FMM) with $\mathcal{O}(N)$
- GMRES solver with Jacobi preconditioner

Fast Multipole Method

- Multipole expansion with respect to center c

$$\frac{1}{|x-y|} = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} R_n^m(y-c)S_n^m(x-c)$$

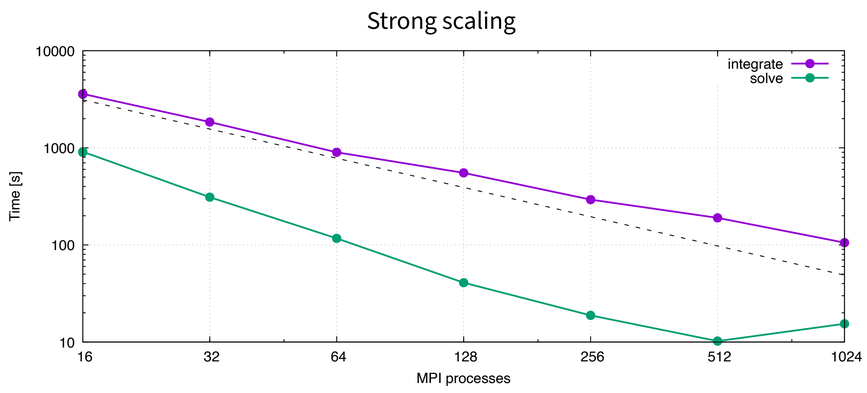
- Regular and singular solid harmonics, R_n^m and S_n^m
- Adaptive multi-level interaction operators

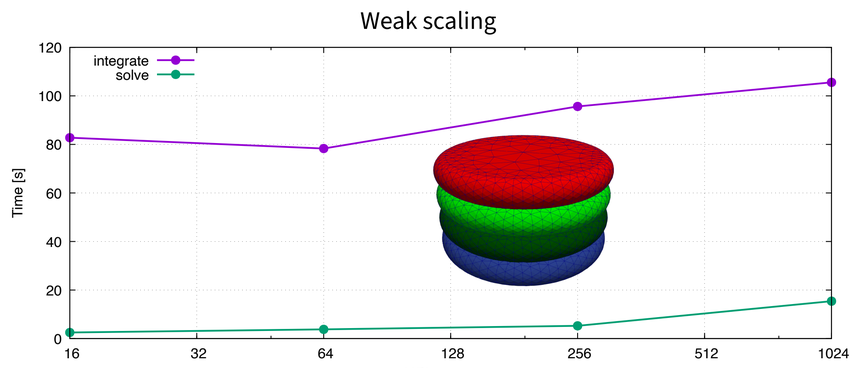


- MPI-parallel and multi-threaded

Scaling Results

- Test model with electrode, two dielectrics and a floating potential
- Up to 3.9M surface elements on up to 1024 MPI processes





Acknowledgments & References

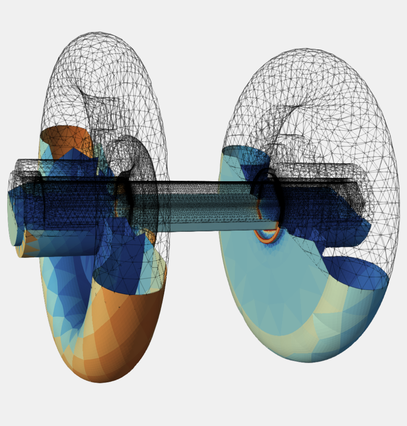
The computational results have been achieved using the Austrian Scientific Computing (ASC) infrastructure. Support in accessing the infrastructure was given by Advanced Computing Austria (ACA), `https://www.asc.ac.at`.

1. A. Blaszczyk, "Region-oriented BEM formulation for numerical computations of electric fields," in *Scientific Computing in Electrical Engineering SCEE 2008*, Springer, 2010.
2. D. Amann, et al. "Simulation of floating potentials in industrial applications by boundary element methods," *Journal of Mathematics in Industry*, 4.1:13, 2014.
3. I. Lashuk et al., "A massively parallel adaptive fast-multipole method on heterogeneous architectures," in *Proc. Conference on HPC Networking, Storage and Analysis*, 2009.

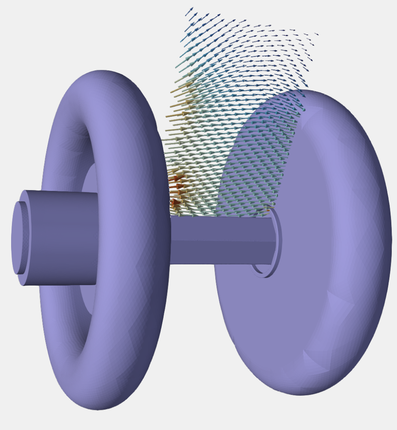
POLARIS — HPC Electrostatics Solver

Features

- Computation of potential and E-field on the surface *and* inside the domain using a FMM-accelerated Boundary Element Method
- Runs on Linux clusters and workstations with MPI



Computed field, 1.6M elements



Field evaluation

More information and free trial version

`https://www.tailsit.com/polaris`