

Fast Boundary Element Methods

THE CURSE OF COMPLEXITY

Boundary Element Methods (BEM) are distinguished by the fact that certain problems can be analysed by using only a surface mesh with the following advantages:

- significantly less time for mesh generation
- no mesh generation for exterior domain problems (e.g., acoustics and radiation)
- accurate incorporation of the exterior domain without truncated volume meshes
- no mesh deformation or re-meshing when subdomains move with respect to each other

A severe limitation of BEM is its inherent complexity: the system matrix is not sparse but fully populated. This implies the computation time and storage costs for this matrix grow quadratically in the number of unknowns. As an illustration, a refine-by-splitting typically yields a 4-fold number of unknowns which then gives a cost increase by a factor of 16. Hence, computing resources are soon exhausted for the analysis considering practically relevant problems. Also, the quadratic complexity is forwarded to the solution of the system of equations: whereas a direct solution becomes prohibitively expensive due to its cubic complexity, even an iterative solution scheme suffers from the quadratic cost of each matrix-vector multiplication.

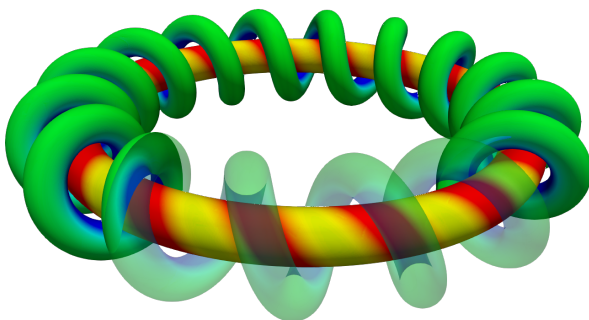
REMEDIES

We present three potential solutions for the complexity problem. All of them make use of a recursive subdivision of the surface mesh, which gives rise to a so-called cluster tree. The three presented approaches are the **Single-Level SVD compression (SL-SVD)**, the **Adaptive Cross Approximation (ACA)**, and the **Fast Multipole Method (FMM)**:

SL-SVD The combinations of leaf clusters which do not lead to a block on the matrix diagonal are compressed with a truncated singular value decomposition (SVD). Clearly, every matrix block has to be first computed in order to be compressed.

ACA Cluster tree node combinations on any level that are considered **admissible** are approximated by a heuristic black-box method. Despite a good complexity, this approach is not robust and can fail completely for several relevant application cases.

FMM The cluster tree gives rise to the notion of near- and farfield. For the latter, a kernel expansion is used to approximate the BEM integrands. This method gives the best overall performance and does not exhibit any robustness problems.



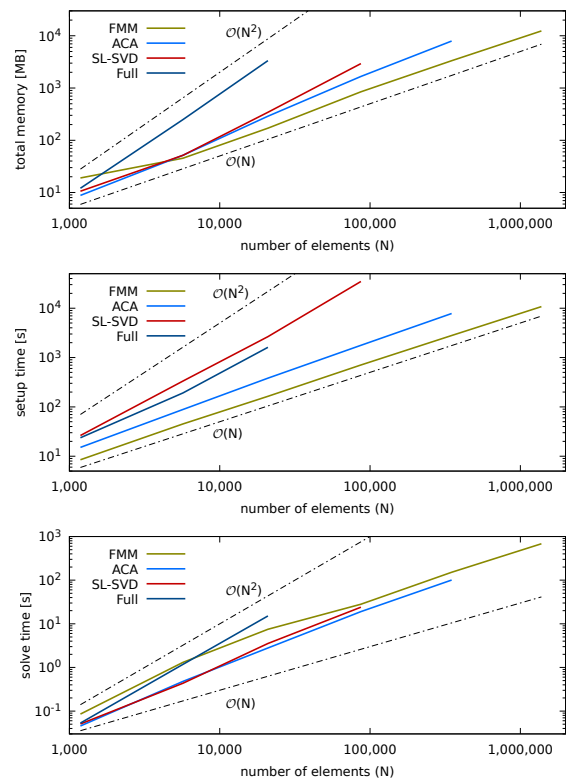
Computed surface charge density

ILLUSTRATIVE EXAMPLE

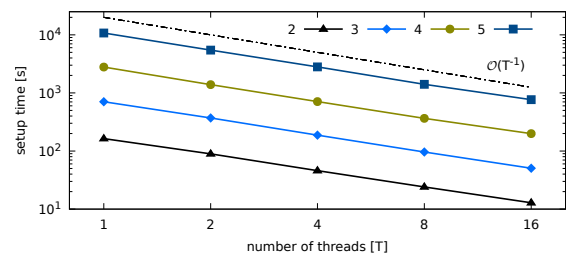
We consider the analysis of an electrostatic problem where two separate surfaces are set to different voltages and the surface charge density is computed. Demonstrated are the performances in terms of storage requirements, pre-solve computation and iterative solution times with an ABPX preconditioner, and the scaling for a multi-threaded FMM analysis.

Level	0	1	2	3	4	5
#elements	1.2K	5.7K	20K	87K	350K	1.4M

Mesh resolutions



Storage costs, setup and iterative solution times for one CPU



Multi-threaded FMM setup time for mesh levels 2-5

TAILSIT GmbH
Nikolaipplatz 4
8010 Graz, Austria
www.tailsit.com
info@tailsit.com
+43 676 974 6416

Contact Person:
Dr L. Kielhorn

TAIL SIT

tailored
simulation
tools

